Numerical Analysis of Behavior of High-viscosity Electromagnetic Fluid Using a Coupled Method of Particle Method and FEM

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Electromagnetic fluid, which is usually a suspension of extremely fine particles of electric or magnetic characteristics in a carrier fluid, shows unique behaviors such as a large deformation in electromagnetic field. This paper deals elongation and sharpening of electromagnetic fluid with extremely high viscosity in electrostatic field. This behavior of fluid is very difficult to calculate with FEM because it includes an intensive deformation and a large change of interfacial surface. Therefore we propose an improved three dimensional combined analysis method of FEM and MPS (Moving Particle Semi-implicit). Electrostatic field and fluid deformation are calculated with FEM and MPS respectively and they are weakly coupled. Specifically electrostatic field and electrostatic force acting on fluid is calculated with FEM in each step and the force is distributed to particles in MPS in the form of electric stress as an external force. This research calculates fluid speed due to viscous term in Navier-Stokes equations implicitly and succeeds in calculating a deformation of high-viscosity electromagnetic fluid which is similar to measured results.

***Index Terms*—Electromagnetic fluid, Coupled method, FEM, MPS, Implicit method**

I. Introduction

Electromagnetic fluid (e.g. ferrofluid, electrofluid) is one of the smart fluids and widely researched for industrial use such as a sealing material or a micropump. This research deals an intensive deformation of electromagnetic fluid with extremely high viscosity due to an electrostatic field in micro scale. To control a fluid shape which has electromagnetic properties can be applied to MEMS devices for example.

Numerical analysis is a very useful tool to understand the mechanism of a deformation of electromagnetic fluid because it is related to a complex combination of electromagnetic force, surface tension, the force of gravity and other external forces. Our group have been proposing a combined method of the particle method and FEM to calculate behaviors of molten metal in magnetic field [1, 2] or a minuscule droplet of water in electric field [3, 4]. These researches succeeded in qualitatively re-creating a deformation of such fluid in electric or magnetic field but didn’t deal a very intensive and shape deformation of high-viscosity electromagnetic fluid which involves in this research.

In this research, a combined method of FEM and MPS was improved to calculate a behavior of electromagnetic fluid with an extremely high-viscosity. When a very small amount of such fluid is put in an electrostatic field, it is grown down and sharpened without dividing into two parts or collapsing due to a contribution of electric force and surface tension and force of gravity. To calculate an intensive deformation of high-viscosity electromagnetic fluid, this research uses an implicit method for viscous term in MPS and succeeded in re-creating the deformation of electromagnetic fluid.

II. Analysis Method

In this paper, only electrostatic field is considered. The electromagnetic fluid is approximated as a conductive fluid and it does not have true electric charge. Therefore, the governing equation is Laplace equation relevant to electric potential ϕ.

$$\nabla^2\phi = 0 \tag{1}$$

Electric stress f_e on fluid surface is obtained by the equation below. ε_0 and E_n respectively stands for vacuum permittivity and electric field on normal direction. [5]

$$f_e = \frac{1}{2}\varepsilon_0 E_n^2 \tag{2}$$

Next, the governing equation of fluid behavior is Navier-Stokes equation and equation of mass continuity below.

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla P + \nabla\cdot(\nu\nabla\mathbf{u}) + \mathbf{g} + \frac{\mathbf{f}}{\rho} \tag{3}$$

$$\frac{D\rho}{Dt} = 0 \tag{4}$$

Here ρ is density, P is pressure, ν is dynamic viscosity, $\mathbf{u}$ is velocity, $\mathbf{g}$ is gravity acceleration. Eq. (3) is described by Lagrange's approach. D/Dt denotes Lagrangian derivative which means a derivative with respect to a moving coordinate system. In this paper, fluid is considered as incompressible fluid so equation (4) is led by equation of continuity.

A discretized equation of viscous term in implicit method of MPS is below [6].

$$\mathbf{u}_i^* = \mathbf{u}_i^k + \Delta t\frac{2d}{\lambda n^0}\nu\sum_{j\neq i}\left(\mathbf{u}_j^* - \mathbf{u}_i^*\right)w\left(\left|\mathbf{r}_j^k - \mathbf{r}_i^k\right|\right) \tag{5}$$

$$\lambda = \frac{\sum_{j\neq i}\left|r_j^0 - r_i^0\right|^2 w\left(\left|r_j^0 - r_i^0\right|\right)}{\sum_{j\neq i} w\left(\left|r_j^0 - r_i^0\right|\right)} \tag{6}$$

Here $\mathbf{u}^k$ and $\mathbf{r}^k$ respectively means velocity and position at time step k. λ is calculated with weighting function w and initial particle number density n^0. Obtained velocity $\mathbf{u}^*$ is not a correct velocity at next step because the effect of pressure is neglected. $\mathbf{u}^*$ is revised by gradient of pressure the velocity $\mathbf{u}^{k+1}$ is calculated in MPS.

III. Analysis Model

Fig. 1 shows the initial setup of electromagnetic fluid and electrodes. The shape of fluid is stable with a half-sphere due to a surface tension and force of gravity. Then a high voltage is applied between two electrodes and the fluid is grown down by electrostatic force. Because the fluid has a high viscosity (approximately several tens of thousands times of water), it is elongated and sharpened without collapsing.

Fig. 2 is an example of FEM mesh model. The number of particles and meshes are approximately 35000 and 150000 for this model respectively.

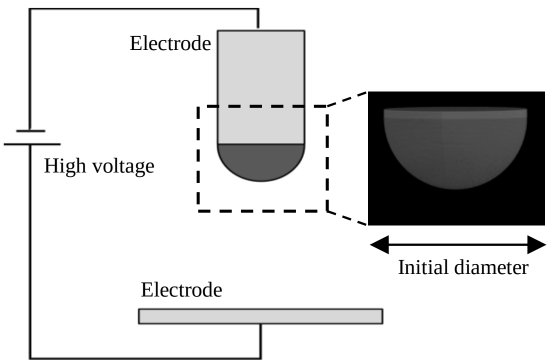

Fig. 1. Setup of electromagnetic fluid and electrodes (MPS model)

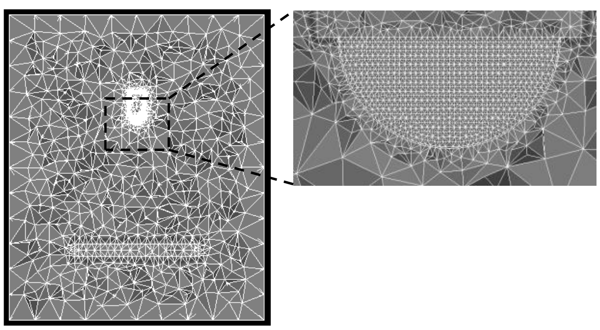

Fig. 2. Example of FEM mesh model

IV. Analysis Results

Fig. 3 shows an initial shape and a deformed shape of electromagnetic fluid. A deformation of electromagnetic fluid due to a contribution of electric force, surface tension and force of gravity is qualitatively re-created. Fig. 4 shows a distribution of electric force of each shape. The length of vector in Fig. 4 is proportional to the largeness of electric force. Therefore it can be seen that electric force is stronger around the tip when the fluid is sharpened. The calculation time of this model was approximately eight hours.

In this result, the number of particles were not enough to calculate more sharpened shape of electromagnetic fluid. This is because a certain amount of particles is needed to represent the shape of the tip in MPS.

(a) initial shape of fluid (b) deformed shape of fluid

Fig. 3. Initial and deformed shape of fluid

(a) initial distribution (b) distribution after deformation

Fig. 4. Distribution of electric stress of Initial and deformed fluid

V. Conclusion

A combined method of FEM and MPS to calculate a large deformation of micro scale high-viscosity electromagnetic fluid in electrostatic field was developed. A deformed shape of electromagnetic fluid was qualitatively re-created.

To calculate a more elongated and sharpened shape, it is necessary to consider an adequate number of particles in MPS. The improved result and comparison with measured results are also going to be reported in full paper.

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